10) a) $\begin{cases} y_1' = y_1 + 6y_2 \\ y_2' = 5y_1 + 2y_2 \end{cases}$

$A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$ $\quad P(\lambda) = \det\left(\begin{bmatrix} \lambda-1 & -6 \\ -5 & \lambda-2 \end{bmatrix}\right) = \lambda^2 - 3\lambda - 28$

Autoval. $\to \lambda^2 - 3\lambda - 28 = 0 \quad \begin{cases} \to 7 = \lambda_1 \\ \to -4 = \lambda_2 . \end{cases}$

Para $\lambda = 7$

$\begin{pmatrix} 6 & -6 \\ -5 & 5 \end{pmatrix} F2 \to 5F1 + 6F2 \begin{pmatrix} 6 & -6 \\ 0 & 0 \end{pmatrix} \to 6x = 6y \to x = y$

$\to \bar{X} = x \cdot \underbrace{(1,1)}_{\text{AUTOVECT. } \lambda = 7.}$

Para $\lambda = -4$

$\begin{pmatrix} -5 & -6 \\ -5 & -6 \end{pmatrix} \to F2 \to F1 - F2 \begin{pmatrix} -5 & -6 \\ 0 & 0 \end{pmatrix} \to -5x - 6y = 0 \to x = -\frac{6}{5}y$

$\to \bar{X} = y\underbrace{\left(-\frac{6}{5}, 1\right)}_{\text{AUTOVECT } \lambda = -4}$ uso múltiplo: $\boxed{(-6, 5)}$

Entonces las soluciones del homogéneo son:

$$\boxed{Y(t) = k_1 \cdot e^{7t} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + k_2 \cdot e^{-4t} \cdot \begin{bmatrix} -6 \\ 5 \end{bmatrix}}$$

6) $\begin{cases} y_1' = -\frac{1}{2} y_1 + \frac{1}{3} y_2 \\ y_2' = \frac{1}{2} y_1 - \frac{1}{3} y_2 \end{cases}$

$A = \begin{bmatrix} -\frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & -\frac{1}{3} \end{bmatrix}$ $P(\lambda) = \det\left(\begin{bmatrix} \lambda+\frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{2} & \lambda+\frac{1}{3} \end{bmatrix}\right) = \lambda^2 + \frac{5}{6}\lambda \cancel{+\frac{1}{6}} \cancel{-\frac{1}{6}}$

$\to$ Autoval. $\to$ $\lambda^2 + \frac{5}{6}\lambda = 0 \to \lambda \cdot \left(\lambda + \frac{5}{6}\right) = 0$ $\boxed{\lambda_1 = 0 \ \text{y} \ \lambda_2 = -\frac{5}{6}}$

Para $\lambda = 0$

$\begin{pmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{2} & \frac{1}{3} \end{pmatrix}_{F2 \to F1 + F2}$ $\begin{pmatrix} \frac{1}{2} & -\frac{1}{3} \\ 0 & 0 \end{pmatrix} \to \frac{x}{2} - \frac{y}{3} = 0 \to x = \frac{2}{3} y$

$\to \bar{X} = y\underbrace{\left(\frac{2}{3}; 1\right)}_{\text{MÚLTIPLO}}$

$\boxed{(2;3) \ \text{AUTOVECT. } \lambda = 0}$

Para $\lambda = -\frac{5}{6}$

$\begin{pmatrix} -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}_{F2 \to \frac{1}{2}F1 - \frac{1}{3}F2}$ $\begin{pmatrix} -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 \end{pmatrix} \to -\frac{x}{3} - \frac{y}{3} = 0 \to x = -y$

$\to \bar{X} = y \cdot \underbrace{(-1, 1)}_{\text{AUTOVECT. } \lambda = -\frac{5}{6}}$

$\to \boxed{Y(t) = k_1 \cdot \overbrace{e^{0t}}^{=1} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} + k_2 \cdot e^{-\frac{5}{6}t} \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$